

Please write clearly in block capitals.

Centre number

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Candidate number

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Surname

Forename(s)

Candidate signature

I declare this is my own work.

A-level FURTHER MATHEMATICS

Paper 1

Time allowed: 2 hours

Materials

- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
3	
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7	
8	
9	
10	
11	
12	
13	
14	
15	
TOTAL	



Answer **all** questions in the spaces provided.

1 Find

$$\sum_{r=1}^{20} (r^2 - 2r)$$

Circle your answer.

[1 mark]

2450

2660

5320

43680

2 Given that $z = 1 - 3i$ is one root of the equation $z^2 + pz + r = 0$, where p and r are real, find the value of r .

Circle your answer.

[1 mark]

−8

−2

6

10



3 The curve C has polar equation

$$r^2 \sin 2\theta = 4$$

Find a Cartesian equation for C .

Circle your answer.

[1 mark]

$$y = 2x$$

$$y = \frac{x}{2}$$

$$y = \frac{2}{x}$$

$$y = 4x$$

Turn over for the next question

Turn over ►



Show that the solutions to the equation

can be expressed in the form

where a and b are integers to be found.

[5 marks]

[illegible]

[illegible]

Turn over ►



The matrix $\mathbf{M}$ is defined by $\mathbf{M} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Prove by induction that $\mathbf{M}^n = \begin{bmatrix} 3^n & 3^n - 1 & -3^n + 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ for all integers $n \geq 1$

[illegible]

[illegible]

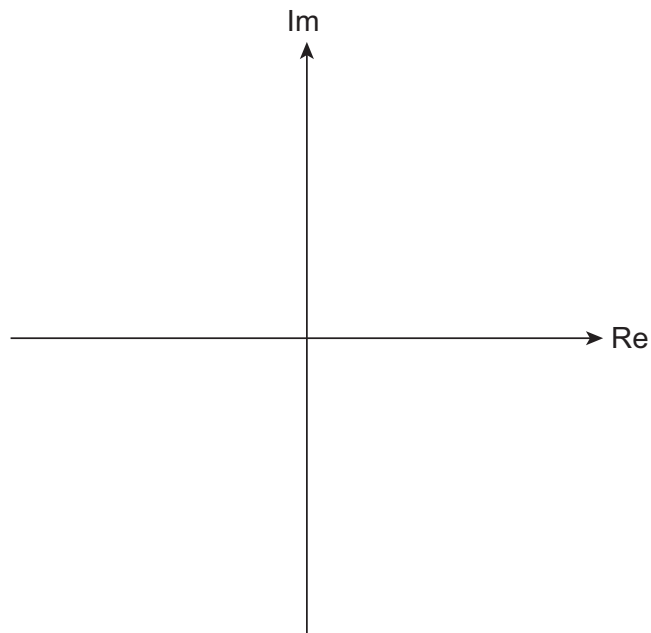
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[7 marks]

[illegible]

- [2 marks]**



- [1 mark]**

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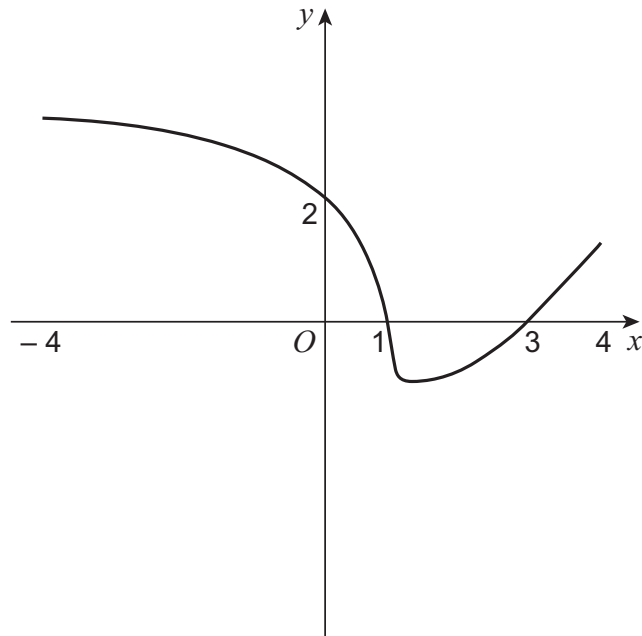
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7 The diagram below shows the graph of $y = f(x)$ ($-4 \leq x \leq 4$)

The graph meets the x -axis at $x = 1$ and $x = 3$

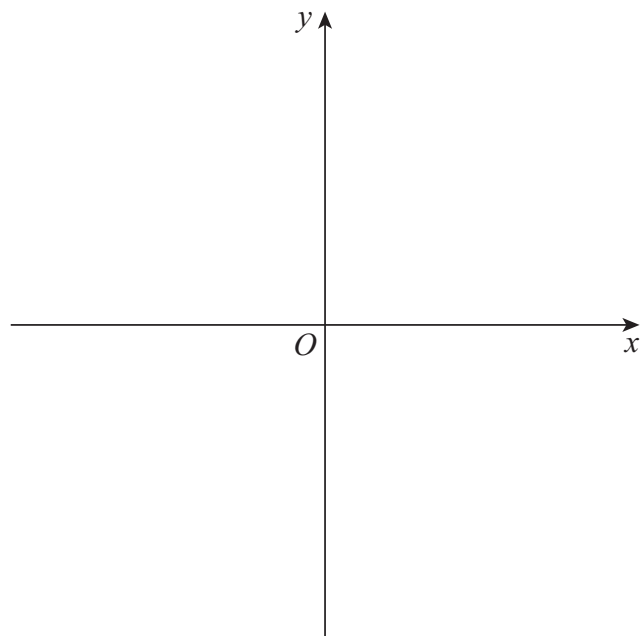
The graph meets the y -axis at $y = 2$



7 (a) Sketch the graph of $y = |f(x)|$ on the axes below.

Show any axis intercepts.

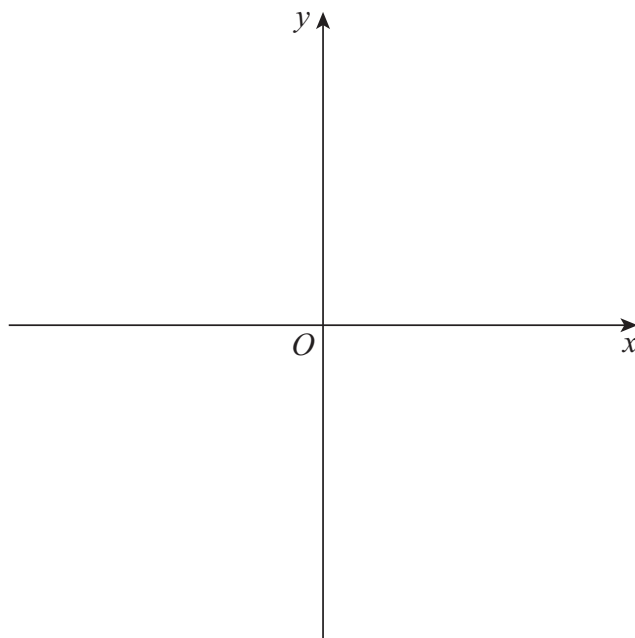
[2 marks]



7 (b) Sketch the graph of $y = \frac{1}{f(x)}$ on the axes below.

Show any axis intercepts and asymptotes.

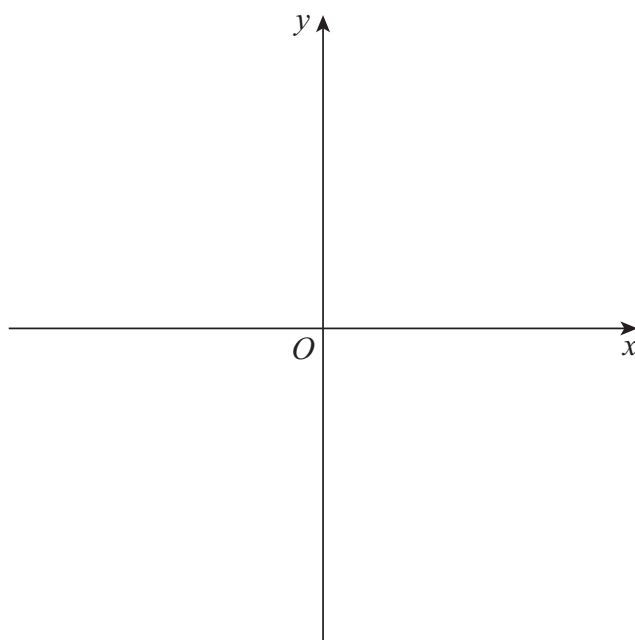
[3 marks]



7 (c) Sketch the graph of $y = f(|x|)$ on the axes below.

Show any axis intercepts.

[2 marks]



Turn over ►



A particle of mass 4 kg moves horizontally in a straight line.

The following horizontal forces act on the particle:

- When $t = 70$, $v = 54$

Give your answer to **four** significant figures.

[6 marks]

[illegible]

[illegible]

Turn over ►



$$\lim_{x \rightarrow \infty} (xe^{-x}) = 0$$

[4 marks]

[illegible]

Evaluate the improper integral

showing the limiting process.

[6 marks]

[illegible]

11 The line L_1 has equation $\mathbf{r} = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$

The line L_2 has equation $\mathbf{r} = \begin{bmatrix} 6 \\ 4 \\ 1 \end{bmatrix} + \mu \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

- 11 (a)** Find the acute angle between the lines L_1 and L_2 , giving your answer to the nearest 0.1°

[3 marks]

- 11 (b)** The lines L_1 and L_2 lie in the plane Π_1

- 11 (b) (i)** Find the equation of Π_1 , giving your answer in the form $\mathbf{r} \cdot \mathbf{n} = d$

[4 marks]



[1 mark]

Find the angle between the planes Π_1 and Π_2 , giving your answer to the nearest 0.1°

[4 marks]

[illegible]

The matrix $\mathbf{A} = \begin{bmatrix} 1 & 5 & 3 \\ 4 & -2 & p \\ 8 & 5 & -11 \end{bmatrix}$, where p is a constant.

Given that $\mathbf{A}$ is a non-singular matrix, find $\mathbf{A}^{-1}$ in terms of p .

State any restrictions on the value of p .

[6 marks]

[illegible]

$$\begin{array}{rcl} x + 5y + 3z & = & 5 \\ 4x - 2y + pz & = & 24 \\ 8x + 5y - 11z & = & -30 \end{array}$$

[4 marks]

[illegible]

[4 marks]

[illegible]

13 The transformation S is represented by the matrix $\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$

The transformation T is a translation by the vector $\begin{bmatrix} 0 \\ -5 \end{bmatrix}$

Kamla transforms the graphs of various functions by applying first S, then T.

Leo says that, for some graphs, Kamla would get a different result if she applied first T, then S.

Kamla disagrees.

State who is correct.

Fully justify your answer.

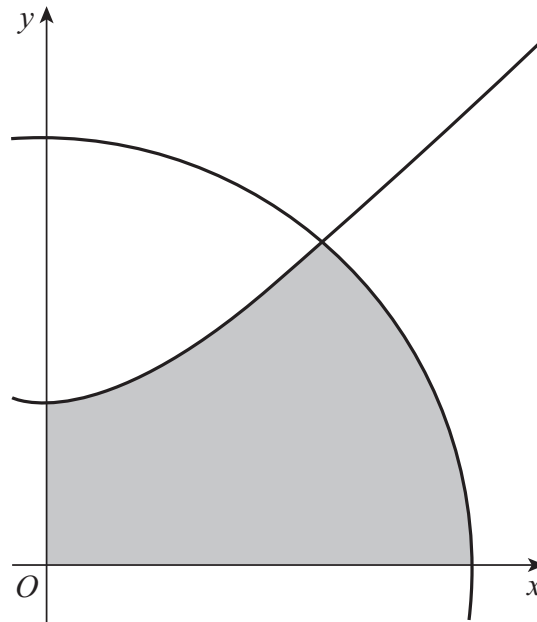
[3 marks]

Turn over ►



The hyperbola H has equation $y^2 - x^2 = 16$

The diagram below shows part of the graph of H and part of the graph of C .


$$\frac{16\pi}{3} + 8 \ln \left(\frac{\sqrt{2} + \sqrt{6}}{2} \right)$$
[illegible]

[illegible]

15 In this question use $g = 9.8 \text{ m s}^{-2}$

A particle P of mass m is attached to two light elastic strings, AP and BP .

The other ends of the strings, A and B , are attached to fixed points which are 4 metres apart on a rough horizontal surface at the bottom of a container.

The coefficient of friction between P and the surface is 0.68

- When the extension of string AP is e_A metres, the tension in AP is $24me_A$
- When the extension of string BP is e_B metres, the tension in BP is $10me_B$
- The natural length of string AP is 1 metre
- The natural length of string BP is 1.3 metres

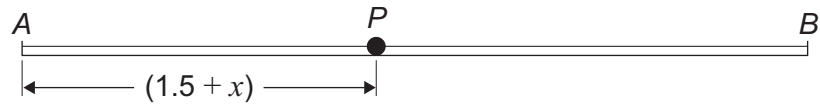


15 (a) Show that when $AP = 1.5$ metres, the tension in AP is equal to the tension in BP .

[1 mark]



At time t seconds after P is released, $AP = (1.5 + x)$ metres.


$$\frac{d^2x}{dt^2} + 34x = 6.664$$

[3 marks]

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Question 15 continues on the next page

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At time t seconds after P is released, the oil causes a resistive force of magnitude $10mv$ newtons to act on the particle, where $v \text{ m s}^{-1}$ is the speed of the particle.

Find x in terms of t when P is moving towards A .

[9 marks]

[illegible]

END OF QUESTIONS



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